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DUSTY PLASMA SAIL FEASIBILITY STUDY

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Introduction

The plasma sail concept uses the same mechanism to generate thrust as the "mag sail" concept, as developed by Robert Zubrin and others (see NIAC web site). What Zubrin and many other discovered, was that the size magnet needed to generate a magnetic sail was either too heavy or made of "unobtainium" superconductors. The insight Robert Winglee realized, was that plasma itself is in some ways an obtainable superconductor that can be controlled by adjusting current, temperature, density, ion species, and central magnet strength. In other words, the 30 years of research into plasma physics could be leveraged to make a plasma sail with current technology. He presented and published [Winglee, 2000] his simulation of a plasma sail as an example of what such a technology would look like and the potential uses it had. There have been several reanalyses of his concept, some favorable, some less so. Our own reanalysis suggests that the concept is feasible but should undergo fine tuning that will require further simulations.

The point of all sails is to intercept momentum, so that the area of the sail is directly proportional to the thrust. For a plasma sail to operate on the solar wind (which is ~1nPa of pressure as compared to 1mPa for sunlight), it must have about 30 times the diameter of a sunlight sail. Since the smallest useful solar sails proposed were ~1 km in diameter, a plasma sail needs to be approximately 30km in diameter to achieve comparable performance. So the first goal of the new plasma technology is to produce a bubble 30km in diameter in the solar wind.

<u>How Big a Bubble?</u>

A vacuum dipole has a magnetic field strength that drops as $1/R^3$. Taking the standoff distance at the nose to be where solar wind magnetic + ram pressure = plasma sail pressure, we get about 50nT for conditions at the Earth's nose. This number will drop considerably as a plasma sail moves out of the solar system, not only because both solar wind magnetic field and ram pressure drop with density ($\sim 1/R^2$), but also because the accelerating sail moves with the solar wind, which reduces the pressure of both. As Winglee points out, this causes the plasma sail to expand as it moves out of the solar system, in such a way as to keep the force constant. Even at the Earth, 50nT is the maximum field, since the flanks of the magnetosphere have much smaller field strengths, nevertheless, a plasma sail will start at Earth orbit, so very conservatively, we can use 50nT for the outer boundary condition. If we set 15km as the radius, and perhaps 1T surface field for the central magnet (presently achievable with NIB magnets), this fixes the radius of our magnet,

$$B = B_0(R_0/R)^3,$$

giving a 55 meter diameter for a 1T magnet. Conversely, if we take a 1m radius satellite (typical rocket farings are 2-3 meters in diameter), then the field strength becomes 170kT, or roughly 1000 times larger than the strongest laboratory magnets ever built. It should be clear why Zubrin's approach was not pursued further. Since a larger diameter magnet would greately reduce the required field strength (e.g. B*A=constant), Zubrin's approach might succeed if the plasma itself took up the role of the large diameter magnet. In common parlance, we say that the plasma has "inflated" the vacuum magnet field.

Now there are two basic ways to inflate a plasma bubble: plasma pressure, and magnetic pressure. (One might envision kinetic ram pressure or electric field pressure as possible inflationary strategies, but the need for an MHD equilibrium reduce all these possibilities to the two above.) As examples, one might argue that laser ablation plasma (Gekelman 2003) or the AMPTE thermite explosions are examples of pure thermal plasma expansion, whereas the Earth's magnetic Dst storm (Chapman 1931) is a pure example of magnetic pressure expansion. Of course, the two types of expansion are related, such that strong currents and plasma waves are generated in a thermally expanding bubble, and plasmasheet plasma is injected in a Dst storm, but for the purposes of this response we consider them separately.

Now there are some fundamental differences between these two pressure terms. Speaking heuristically, magnetic pressure is non-local, depending on current systems from the entire domain, whereas plasma pressure is local, satisfying local hydrodynamic equilibrium. A full self-consistent description will require careful modelling, but we can make a few simplifications that should give us ballpark limits.

Plasma Pressure

Since instabilities tend to occur whenever the plasma pressure dominates over the magnetic pressure, we assume that for a plasma sail to maintain integrity long enough for momentum transfer, it will operate in a regime of $\beta\sim1$. One might, of course, attempt stabilization techniques such as those used in tokomaks, but the selling point of a plasma sail is that it uses simple technology that is presently available, which would suggest that thermal pressures much exceeding $\beta=1$ are unlikely. Since the plasma sail is expanding against a constant solar wind pressure, a new equilibrium is reached when we add in the plasma pressure. Using our RC self-similar model below, we assume that the magnetic field outside the RC region scales as $1/R^3$. So if we include the plasma pressure along with the magnetic pressure, the energy density will double, but the radius will not. There are two equivalent ways to say this, either the standoff magnetic field will drop to 50nT/$\sqrt{2}$, or the bubble will expand to a larger diameter. Since our calculation uses the standoff field for all models while holding the radius constant, this has the effect of reducing the required magnetic field strength by 30%. While significant, this hardly changes the order of magnitude of this calculation, indicating that the whole plasma sail concept hinges on whether magnetic pressure is sufficient to inflate a bubble to 15km radii.

Magnetic Pressure

A bar magnet, immersed in a plasma, will exclude the plasma and form a bubble (Chapman 1931). But as we saw above, the vacuum dipole field diminishes too rapidly. Using both the Earth's and Jupiter's magnetospheres as a guide, we argue that injecting plasma into a vacuum dipole field creates currents that expand the bubble diameter so that Earth's field inflates 10-20% during a magnetic storm. Likewise Jupiter's field is inflated 300-400% by Io's injected plasma. That is, Jupiter's standoff distance without plasma would be around 35 jovian radii, whereas Voyager and Pioneer have observed the magnetopause at 120 R_J. The Earth's inflationary field is caused by the ring current, 30-300keV ions located between 3-5 R_E from the Earth. The source of this plasma normally the plasmasheet, which in usually has only weak diffusive access to the ring current except during storms when large electric fields cause direct injection (e.g. Sheldon 1993). Jupiter's plasma source is thought to be Io's volcanos that inject directly into the magnetosphere at 6 R_J. In both cases, the injected plasma rapidly condenses at the dipole equator, forming an equatorial ring current that is primarily responsible for inflating the field.

Note that the Earth's ring current plasma can be many R_E from both the earth and the magnetopause yet still exert a magnetic pressure of inflation, a consequence of the non-local nature of this pressure term. This is also true of Jupiter's magnetodisk, and is expected to be true of Winglee's mini-magnetosphere. That is, much of the plasma injected into the vacuum dipole bubble will redistribute itself into a ring current and provide magnetic pressure that inflates the bubble. Therefore we model this component of the inflation with a ring of current, or in the case of Jupiter, a series of rings that approximate a disk. (A direct integration of a current sheet distribution produces an analytically less tractable form than a sum of rings.) We interpret Winglee's simulation results from his 2000 JGR paper, as well as our own kinetic simulations, as supporting a Jupiter-like magnetodisk, where the magnetic field strength in the dipole magnetic plane diminishes with 1/R dependence. This is strictly a consequence of having a semi-infinite magnetodisk, or a sum of current carrying rings. We recognize, of course, that at the inner and outer boundary, fringing of the fields will cause a faster decay of the magnetic field, smoothly interpolating to the vacuum dipole solution.

Using this simulation result in its simplest form, we can now recalculate the required magnet size using the relationship,

$$B = B_0(R_0/R)$$

As before if we use a 1T magnet to create a 15km bubble, the required magnet diameter is an astonishing 0.5mm. Conversely, if we use a 1m radius magnet, we need only a weak 52mT field strength. This result appears too good to be true, nature is never so kind. What physics did we overlook?

There are two places that we have perhaps ignored some crucial data. First, at both Jupiter and Earth, the ring currents exist only over a small radial region of the entire bubble. Thus the 1/R scaling strictly applies only within this ring current region, and $1/R^2$ or $1/R^3$ scaling applies outside. Hints of this were observed in our own kinetic simulations of the plasma sail. Second, at both Jupiter and Earth, the flux in the ring current never exceeds the flux in the central magnet. There are good physical reasons for these observations related to the stability of such plasma ring currents, which is what we explore next.

Ring Current Cartoons

The ring current stability question is prone to misunderstanding, so we attempt to provide several heuristic cartoons in the hope that we have caged the elusive physics.

Cartoon 1: Imagine that we have laid a loop of fine gauge copper wire on a slick countertop, and pump an ampere of current through it. In addition to getting hot, the wire will billow out into a circular shape. This is because of the repulsion of oppositely directed currents on opposite sides of the loop. Alternatively, the magnetic flux threading the interior of the loop is compressed relative to the flux outside the loop and exerts a magnetic pressure. In a like fashion, the Earth and Jupiter's RC will also expand outward. Two things constrain the unlimited expansion of the RC, the pressure balance at the outer boundary, and the tension force from the inner magnet (which can also be stated as the need to conserve encircled flux). Now if one imagines that the magnetosphere is a water-filled balloon, it is clear that compressing it between one's hands is an instability prone process, whereas the tension force of the latex (or conservation of water volume minimizing the area) is a stable process. That is to say, it is the central magnet that provides the stable confinement of RC, not the solar wind pressure. Even if we were to remove the solar wind (e.g., a DC-glow discharge around a laboratory magnet) the RC would not expand beyond a few magnet radii. Thus if the RC were to produce much more flux than the central magnet could contain, it may not be stable, since the internal constraint would be less important than the external constraint.

Now let us replace the single loop of wire with a three stranded loop and drive an ampere through the strands, what would happen? The 3 strands would expand as before, but in addition, the strands would coalesce because of the mutual attraction of like currents. Thus in the absence of additional forces such as exist at Jupiter, one might expect that the RC would form a single band or current system in force balance with the central magnet tension, and the self-induced expansion.

Cartoon 2: If we imagine that we start with a permanent magnet for the central magnet, and lay a loop of wire some distance away from it, and gradually ramp up the current in the wire loop, what are the forces on the system? As is well known, two magnets repel when their dipole moments are anti-parallel, attract when they are parallel, and experience a torque when they are not parallel. The interaction between loops and magnets is not so intuitive, but perhaps can be reasoned out. Since energy is proportional to B^2, the magnet and loop will be at an energy minimum when their external fields cancel. This means that the permanent magnet will be aligned with the magnetic moment of the loop, and move toward the center of the loop, so that the external fields of the permanent magnet will cancel the field inside the loop. One might imagine the analogous operation of the starter solenoid on an automobile.

Let's test this conclusion with the Earth. The dipole moment of the earth points south, the external field points north at the equator. The RC moves counter-clockwise around the Earth, such that the RC field is southward for all regions inside the RC. Hence the external dipole field cancels the RC field producing a dip in |B| during a Dst storm. That works.

Cartoon 3: Now consider what happens when the RC is made stronger and stronger. At first field strength inside the RC but outside the magnet (Dst) decreases, but at some point, the field would completely reverse direction and grow stronger. Since the external dipole magnet field is northwards, there must be a null boundary, a separatrix between the fields of the RC and the fields of the central magnet. Inside that separatrix, the central magnet is disconnected from the RC field, looking something like a plasmoid, and as the external magnetic pressure is increased, this plasmoid would experiences a repulsive force. Again, this is analogous to a UMd theta-pinch demonstration, in which an empty coke can is placed in a ten-turn coil of 10# wire, through which is dumped the output of a 1 F, 5000V capacitor. The can is ripped in half and forcibly expelled from the device.

Cartoon 4: If we take the above example, turn off the RC, and extend an infinite plane through the equator of the dipole magnet, the integrated flux outside the magnet must equal the flux passing through the center (that's $\nabla * B=0$). If carry out the definite integral, say from the surface of the magnet to twice the loop radii, we will have a constant value. Now as the current in the loop of wire is ramped up, the initial effect on the integrated flux density is nearly neglible, because any contribution from flux inside the ring is balanced by an opposite contribution outside the ring. The loop magnetic moment is aligned with the magnet, so there must be a net increase if our outer boundary were at infinity, but the additional flux is being added outside our outer boundary. Another way of saying this, is that the RC does not add flux in the vicinity of the central magnet, it merely redistributes it.

Obviously this approximation breaks down when the RC carries more flux than the initial magnet, but first let's look at the redistribution. The field lines from the central magnet that extend beyond the RC are stretched outwards, whereas field lines inside the RC are not so much redirected as weakened, so that magnetic tension force pulls in the ring, and magnetic pressure is reduced inside the ring, also enhancing the pull. This redistribution changes the radial profile of the field such that it goes from a vacuum $1/R^3$ to a magnetodisk 1/R, but it can only do this as long as there is flux to redistribute. Once the RC flux is comparable to the magnet, the solution reverts to that of a pure current ring, the central magnet field lines no longer encircle the ring current, but close in on themselves, losing both the tension outside and the pressure inside. And the definite integral grows linearly with increasing current.

If our heuristic reasoning is correct, then the RC can carry only an amount of flux roughly comparable to the central magnet before instabilities set in. This inflates the magnetosphere more than a factor of two, however, because the RC is at a much larger radius than the central magnet. Since the placement of the RC is critical to this calculation, which is itself constrained by the central magnet, it requires a self-consistent approach not presently at our disposal, and we cannot do much better than order-of-magnitude in this calculation. Therefore we address a related, but more tractable question: what are the stability criteria for a central magnet trapped in the field of a RC?

Magnetic Stability of Ring Current

We now attempt to derive the heuristic reasoning above with more mathematical rigor.

Dipole Stability in RC field

From Jackson, we can write that the force between a dipole magnet with magnetic moment, **m**, and a magnetic field is,

$$\mathbf{F} = \nabla(\mathbf{m} * \mathbf{B})$$

Since we have already assumed that the torque is zero and the magnet is aligned with the current ring, we can write this,

$$\mathbf{F} = m \nabla B_z$$

where B_z is the z-component of the RC magnetic field. Unlike most books, which take the limit at the center, we keep both radial (ρ in cylindrical coordinates) and z-components of B_z, and derive the following:

$$F_z = [-3m\,\mu / k^{5/2}]\ z\,(2a^2+2z^2 - 3z\rho^2 - za\rho)$$
$$F_\rho = [-3m\,\mu / k^{5/2}]\ (3a^3+3az^2 + a\rho^2 + 4\rho z^2 + 5\rho a^2 - \rho^3)$$

where m is the magnetic moment of the magnet, μ is the magnetic moment of the RC, $k=(a^2+z^2+\rho^2 + 2a\rho)$, and "a" is the radius of the RC. Note that for $\rho < a$ (in the radial direction) and $z\rho<a^2$ in the z-direction, the force is negative, indicating unconditional stability.

RC Stability in a Dipole Field

By symmetry, this ought to give the same answer as the above section, but as check on the approximation used for the RC field, we carry out this integral. Here we use Ampere's Law,

$$F = -I \int B_d \times dl$$

Since at the dipole equator, the field is purely in the z-direction, and the current is purely in the φ-direction, the resulting force is pure F_ρ, which when integrated around a circle, is zero. So this is an equilibrium point. In order to calculate the stability, we need to show that the force obeys Hooke's Law in both directions, it is always a negative restoring force.

As before, we assume the torques are zero, so we displace the RC perpendicular to the dipole plane to get the z-component of the force, and in the plane to get the x-component.

$$\mathbf{F} = \nabla\, \mathbf{(m * B)} = \nabla\, (mB_z) = m\, \mu/(\rho^2+z^2)^{3/2}$$
$$Fz = \int di \times B(a,z) = Im(-3\mathbf{z})/(a^2+z^2)^{3/2}$$
$$Fx = \int di \times B(a+\mathbf{x},0) = Ima^2\, (-2\mathbf{x})/(a^2+x^2-2ax)^{5/2}$$

Thus a ring of current is unconditionally stable around a dipole field.

RC Stability in a RC Field

But when the central magnet cannot be described as a point magnetic dipole we have to carry out a volume integral to estimate the force between them. We would like to model the finite size of the central magnet as a loop of current of radius b inside the RC of radius with a force due to the interaction between $B_{RC}(a)$ and $B_{RC}(b)$. Unfortunately, the expression for the magnetic field or vector potential of a RC given in Jackson, is either in terms of elliptic integrals, spherical harmonics or Bessel functions. The series expansion for elliptic integrals is not valid outside the RC, nor is the asymptotic expression derived from it particularly accurate in this area. The spherical harmonics are much better, but converge very slowly in the region around the RC.

Thus the stability question must be resolved numerically. Work done on diamagnetic cavities in the Earth's dipole (Sheldon 2001) indicate that a finite sized central magnet will produce a central plasmoid that grows increasingly unstable to z-axis perturbations as it occupies more volume, but should converge to the above results when the inner ring shrinks, a<<b.

RC radius stability

Having ascertained that a RC as a rigid body is stable in the presence of a dipole (and can therefore transmit thrust), the last question to be addressed is the force-free condition for RC radii. If we consider an element of ring current dl, the two opposing forces are the ixB attractive force of the central magnet, and the self-repulsive force of the remainder of the current ring. The attractive force is straightforward:

$$F_d = i\, B_d(\rho=a) = \mu\, i\, /a^3$$

directed inward, where μ=the magnetic moment of the central magnet.

The self-repulsive force is a bit harder to calculate. A straightforward calculation of the Biot-Savart law doesn't converge when the current element approaches the origin. Instead, we use the RC magnetic field solution at R=a to calculate the B-field at the location of the wire. Again, this goes infinite for a infinitely thin current ring, but since the current is not infinitely thin, it is possible to approximate it with an average of [B(r_inside) + B(r_outside)]/2. Jackson gives an expansion of the B-field due to a ring in terms of associated Legendre polynomials that is valid both inside and outside the ring (unlike the expansion of elliptic integrals). When we take the average of the inside and outside in the limit that $R \rightarrow a$, the expression converges very slowly to an average B-field equal to the B-field in the center of the ring. The force then becomes:

$$F_{RC} = i\ m\ /a^3$$

directed outward, where m= magnetic moment of the ring current. Equating these two forces on a current element, i, we have:

$$\mu = m$$

Using the definition of $m = I\ \pi\ a^2/c$, we have, and assuming that the central magnet is some sort of solenoid of radius s, we have:

$$B_0 s^2 = B_{RC} a^2$$

$$B_0/B_{RC} = (a/s)^2$$

This relation tells us that the field inside a ring of current that is stably encircling a central magnet in the absence of plasma, must follow a $1/R^2$ law.

Magnetic Pressure: RC Contribution

In the plasma-free volume between the ring current and the central magnet, the B-field magnitude decreases as $1/R^2$, rather than a vacuum dipole worst case of $1/R^3$. This $1/R^2$ dependence also appears consistent with our simulations. Using the Earth's ring current as a paradigm, we might argue that the magnetic field goes as $1/R^3$ from 10-5 R_E, and then as 1/R from 5-2.5 R_E, and back to $1/R^2$ from 2.5 -1 R_E. This self-similar scaling is awfully suggestive, so we use it without much more justification for the mini-magnetosphere. Here is our model:

Region \ Quantity	Radius	Magnetic field
Magnet R_0, B_0	~1 m	~1 T
Inner edge of RC, R_1, B_1	$R_3/4$	
Outer edge of RC, R_2, B_2	$R_3/2$	
Standoff distance, R_3, B_3	15 km	35nT

Applying these boundaries, and using $1/R^2$ between the magnet and the RC, 1/R within the RC, and $1/R^3$ outside the RC, we get the following formulae:

$$B_3 = B_2(R_2/R_3)^3$$

$$B_2 = B_1(R_1/R_2)$$

$$B_1 = B_0(R_0/R_1)^2$$

which can be combined to estimate either B_0 or R_0. If we use R_0=1m, then the 3 expressions can be combined to produce:

$$B_0 = B_3\ R_1 R_3^3\ /\ (R_0^2 R_2^2) = B_3\ R_3^2\ /\ R_0^2$$

Substituting in the values from the table. Assuming R_0 = 1m (typical rocket farings are 2-3m diameter), then B_0~7.8T. Conversely, putting in 1T in the expression for B_0 (NIB magnets have this magnetic field strength) retrieves 2.8 m for R_0. Or we can put in both 1T and 1m and solve for R_3=5.3km.

This is getting closer to what we intuitively expected, though it is a bit larger than we would prefer to fly. For the level of sophistication of this calculation, it is in the same ballpark as Winglee's estimate.

With Thin RC

Some debate surrounds the assumption of 1/R scaling within the RC. If we argue that the attractive force of parallel currents (theta-pinch) reduces this region to a wire, then we have collapsed R_1 and R_2, leading to the equation:

$$B_3 = B_2(R_2/R_3)^3$$
$$B_2 = B_0(R_0/R_2)^2$$
$$B_0 = B_3 R_3^3/(R_2\, R_0^2)$$

If we set $R_2 = R_3/2$ as before, then R_0=1m, B_0=1T results in a bubble of radius 3.8km. That is, even without an extended region of 1/R ring current, bubbles of 4km radii are to be expected.

Note that this calculation essentially collapses the RC to a single current. Neither Jupiter nor Earth have such a concentrated RC, suggesting that diffusion, or centripetal acceleration, or plasma instabilities such as flux-tube interchange might conceivably play a role in the thickness of the RC, which would give an answer more similar to the original calculation up above, with 1/R scaling within the RC.

With Thick RC

Finally, since we do not include the plasma physics that might result in a thick RC, we estimate the size of the bubble should the RC be very extensive. We use $R_2 = 0.9R_3$, and $R_1 = 0.1R_3$, with the same 1T and 1m sizes used before. We achieve R_3 = 15.2 km radius, essentially the same size claimed by Winglee. One difference, however, between our solutions, is that we have used a magnet 10 times larger diameter, and 10 times stronger surface field strength. Nevertheless, these order-of-magnitude calculations suggest that if the magnetic flux were distributed throughout the volume, it is possible to create large bubbles.

Magnetic Flux Conservation

There has been some concern that such a 1/R scaling violates flux conservation. The key to this paradox is to recognize that magnetic flux is an inner product of the magnetic field vector and the area normal. A ring of current contributes negative flux in the interior, and positive flux exterior to the ring. Thus starting with a vacuum dipole bubble, calculating the net flux exterior to the magnet will give a constant value. Then inserting a RC and turning up the current will initially produce no net change in the flux, but will increase the diameter of the bubble. Again, this is because the flux is a vector inner product of B with the area, or an integral over the projection of a vector, whereas the magnetic pressure is a scalar, so they are not proportional to each other.

What is the Mechanism of the Momentum Transfer?

This is actually a three part question, answered with different means. The first part is what is the force exerted by the solar wind on the magnetic bubble. The second part is how that force is transmitted from the plasma to the spacecraft. Complicating this further, our proposal suggested that dust suspended in the plasma (dusty plasma) could intercept sunlight momentum and add an additional force to the plasma.

Solar Wind Momentum

The first part, how solar wind momentum is intercepted by the bubble, is complicated because the bubble is comparable in size to the ion gyroradius in the solar wind. If the bubble were much larger than this, then one could compute simply the solar wind pressure and the area of the bubble, but for small bubble sizes, the ions "slip" past the bubble without transferring much momentum. This is analogous to the problem in optics when a scatterer is comparable to the wavelength of light, because geometric considerations are no longer sufficient to describe the interaction. As Khazanov (2003) has shown, the bubble starts to behave geometrically as an opaque obstruction only when its diameter exceeds the about 50km.

Heuristically, when the bubble is smaller than an ion gyroradius, one can view the solar wind ions as unshielded, (ions, not electrons, carry the momentum) interacting with a bare magnet. In this case, the Lorenz force, F = qv X B, causes the ions to deflect sideways if the dipole is oriented perpendicular to the velocity. This change of direction is not necessarily a bad thing, since this is exactly the direction a Δv rocket firing is directed to change orbit characteristics of a spacecraft, but in addition to direction, the magnitude of the force also diminishes. One could integrate an analogous Rutherford scattering law for a magnetic force, but the calculation would be in the unshielded limit, and the actual interaction lies between the two extremes. MHD theorists regard this interaction as a "viscous" interaction, which again, requires a full kinetic simulation to achieve better than order of magnitude estimates. Our own simulations suggest that perhaps 10% of the geometric force is transmitted to the plasma in this regime.

Solar Sunlight Momentum

As our experiments and proposal have shown, it is possible to suspend 3μ sized SiO_2 dust grains in a magnetized plasma, and levitate them against the force of gravity in the laboratory. The laboratoary gravitational force is much greater than the calculated solar photon force, indicating that these dust grains could also be stably trapped in sunlight. If they could be likewise trapped and suspended in the plasma sail, they would contribute a force to the plasma in addition to the solar wind force. Since the solar photon pressure is 1000 times larger than the solar wind pressure, a dust ring component need only be 10% of the area and 1% opaque to double the (assumed) geometrical force on the sail. In the laboratory we were able to achieve densities much higher than this, though admittedly in a very different plasma environment. The field of dusty plasmas is young, and whereas models and simulations have been developed, none have addressed the parameter regimes such a sail would operate at. Indeed, many theorists did not think it possible to levitate dust magnetically. The answer, apparently, is that they are trapped electrically to the plasma, and the plasma is trapped magnetically to the magnet.

Recognizing that without further research into the physics of this phenomenon, all our conclusions are extrapolations, we can nonetheless estimate a force from the dust component of the plasma sail. Our laboratory dust may have had several thousand charges per grain, which is a consequence of a very hot electron plasma. If we assume a photoelectron charging source, this number is closer to several hundred charges per grain.

Since the plasma neutralizes the dust, the plasma density must be greater than the dust density by this amount, or

$$Q_d \times n_d < n_i$$

Winglee estimates densities of $n_i \sim 10^{12}$/cc from a helicon plasma source, which we argue must decrease by at least a factor 1/R from the source (since plasma is confined to a 2-D sheet). Taking perhaps 0.1m for the diameter of the helicon, and 5 km for the outer edge of the RC, which is where we observed our laboratory dust ring, the plasma density is down to about 2×10^7/cc. This gives a maximum dust density of about 10^5/cc. Estimating the thickness of dust sheet to be approximately the magnet diameter of 1 m, gives us a column density of $10^{11}/m^2$. Since the mass of a dust grain goes as the cube, whereas the area goes as the square, flight dust will likely have r <1μ to maximize area to weight ratio. Using this dust, our cross sectional area is:

$$\pi(500\ nm)^2 * 10^{11} m^2 = 0.08 m^2 \text{ or } 8\% \text{ opacity.}$$

The area of the dust sheet is hard to estimate from the laboratory, though it seemed it was about 10% of the radial extent of the laboratory RC+magnet, located at the outside of the visible RC. Using this number we estimate the dust ring to extend from 5-5.5 km. Thus the area of this dust ring is

$$(5.5^2-5^2)/15^2 = 2.3\%$$

Combining these figures gives 8% x 2.3% * 1000 photon/wind = 190% boost to the geometric thrust of a plasma sail.

Likewise, the mass of this dust sail can be estimated as the volume of the sail times the number density and mass per grain. The volume is 5.2 Mm^2 x 1m thick, the density is 10^{11} grains/m^3, and the mass is $4/3\ \pi(500\ nm)^3$ x 1g/cc density (carbon dust) = 260 kg for the sail. The effective area compared to a mylar sail should probably be reduced by the 8% reflectivity, so we give an effective mass loading of 260kg/5.2*0.08 Mm^2 = 0.6 g/m^2. Then the sail looks very attractive without considering the plasma container needed to deploy it. Given the conservative estimates and the level of understanding of the physics, we might easily be an order of magnitude off, but clearly, this is an area that can benefit from further research.

Momentum Transfer to the Spacecraft

There have been some concerns that the plasma is not magnetically well connected to the spacecraft, and therefore cannot transfer momentum to the spacecraft. As we addressed in our first section, the magnet is unconditionally stable in a pure RC field as long as the magnet stays "inside" the ring. Likewise a RC is unconditionally stable in a pure dipole field. The "disconnected" topology only exists when the RC is overdriven, and the central magnet appears as a plasmoid compressed in the center of the RC. Under these conditions, the spacecraft is highly stable in the equatorial plane, but unstable along the axis, analogous to a watermelon seed that is squeezed between the thumb and finger. Two obvious solutions to this instability are either not overdriving the RC, or applying the thrust in the equatorial plane of the dipole.

This second solution is not amenable to a dusty sail, however, since the edge-on dust sail has very little area (though it will be completely opaque). Nor does it seem advisable to fly a first generation plasma sail that has inherent instabilities, so it seems again we would argue that the feasibility of the plasma sail hinges on its ability to find an equilibrium RC geometry that is not overdriven. Once again, this is a self-consistent calculation that is somewhat beyond the capabilities of this order-of-magnitude calculation.

In summary then, it appears that a plasma sail on the order of 15 km will generate only 10% of the geometric thrust expected from the solar wind, but 190% can be extracted from the photon flux. To put a number on it, this is,

$$2\,\pi\,(15000\mathrm{m})^2\,(1\mathrm{nPa}) = 1.4\ \mathrm{N}$$

If then the dust weighs 260kg, and the spacecraft, magnet and plasma source weigh an additional 500kg, we have a 760kg spacecraft with 1.4N of force and an acceleration of 0.0018 m/s^2.

Comparing this to a standard solar sail where we calculate a mass loading for the entire s/c, we have:

$$\text{Pressure/acceleration} = \text{mass/area} = 10^{-6}\ \mathrm{Pa} / 1.8\mathrm{mm/s}^2 = \sim 0.5\mathrm{g/m}^2.$$

This is clearly state-of-the-art in solar sails.

The missions enabled by this sail include trips to outer planets, of course, but since the solar sunlight pressure drops as $1/R^2$ for these trips, solar sails look better in near Earth orbit. Two applications would be a “polesitter” that looks down on the poles of the Earth, and an upstream monitor of the solar particle environment (storm monitor) that corotates with the Earth slower than Keplerian velocity. The polesitter distance R is roughly where the sunlight force balances the Earth’s gravitational force, or $R = (mg/F)^2 = 73$ Re which is better than L1 at 250 Re. Or the solar storm monitor, balances the solar gravitational force minus the solar photon force with centripetal acceleration resulting in $R = (1 - F/m\omega^2 Re)^{1/3} = 0.88$ AU. Again, an ion engine would not be able to provide this sort of thrust, and its equilibrium point would be much closer to the Earth, giving much less lead time should solar weather turn nasty.

Losses from the Mini-Magnetosphere in the Simulation

Although others may address the plasma experiments, our expertise is with dusty plasma experimentation. For these experiments, there have not been any simulations performed to my knowledge. We are presently working on a PIC simulation of the DC glow discharge, and attempting to calculate the space charge that accumulates at the dipole equator. Direct experimental measurement of this space charge using a floating dipole probe (see Sheldon, AGU 2002 talk) gave inconclusive results of about a volt superposed on a noise signal of about a volt. We are also attempting to upgrade the diagnostics to measure this space charge with better signal to noise ratio. Both of these are ongoing research efforts that are highly manpower limited, but perhaps will return results in 6 months to a year.